Network Theorems & Impedance Functions:-

On Characteristic Eqn:

Consider a 2^{nd} order homogeneous diff. eqn. with constant coefficients:

$$a_0 \frac{d^2 i}{dt^2} + a_1 \frac{di}{dt} + a_2 i = 0 \quad —(1)$$

→ soln to this diff. eqn must be such that the soln itself, its 1st derivative & its second derivative, each multiplied by a constant coeff. add-up to zero.

This is possible when, all 3 terms must be of the same form, differing only in their coeff's.

→ The fn always leads by search to exp,

$$i(t) = ke^{st} \quad —(2)$$

where k & s are constants which may be real, imag, or complex.

Substg. (2) in (1), we have

$$a_0 s^2 ke^{st} + a_1 ske^{st} + a_2 ke^{st} = 0$$

$$\Rightarrow a_0 s^2 + a_1 s + a_2 = 0$$

This eqn is cht. or auxiliary eqn.

s is a bi-valued root of cht. eqn.

& is the complex freq.

B.Tech. ELE :-

Transform Impedance / admittance & transform Ckts :-

Xform Impedance of ckt. elements namely R, L & C.

(a) Resistance :-

for an R,

$$u_R(t) = R \cdot i_R(t)$$

~~or $i_R(t) = G \cdot u_R(t)$~~

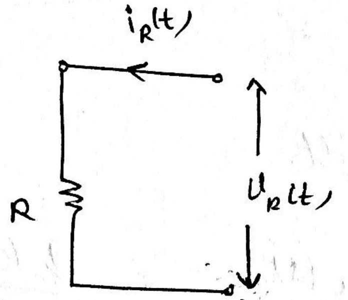
Transform eq ~~Impedance~~ is

$$V_R(s) = R \cdot I_R(s)$$

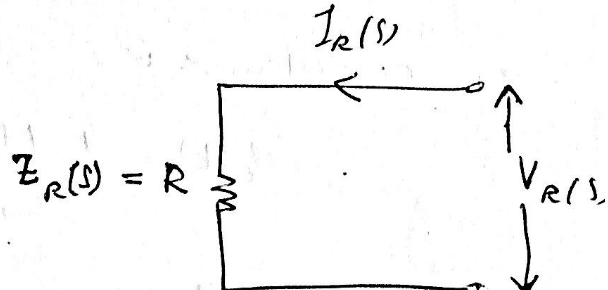
Xform impedance of an R is given by :

$$\boxed{Z_R(s) = R = \frac{V_R(s)}{I_R(s)}} \Rightarrow Y_R(s) = G = \frac{1}{R}$$

→ Thus an R is insensitive to even the complex frequency.



(a) R in Time-Domain



(b) Transform representation of resistor

(b) Inductance :- for an L,

$$v_L(t) = L \cdot \frac{d i_L(t)}{dt}$$

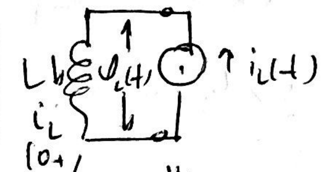
or

or $V_L(s) = L[sI_L(s) - i_L(0+)]$

if $i_L(0+) = 0$, then

$$\boxed{Z_L(s) = \frac{V_L(s)}{I_L(s)} = Ls} \Rightarrow \boxed{Y_L = \frac{1}{Ls}}$$

* llly, from $i_L(t) = \frac{1}{L}\int v_L(t)\,dt$ we have



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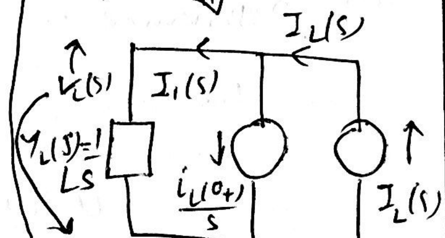

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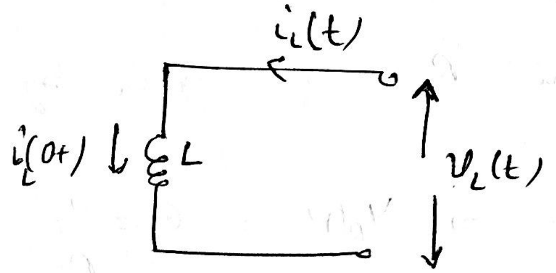
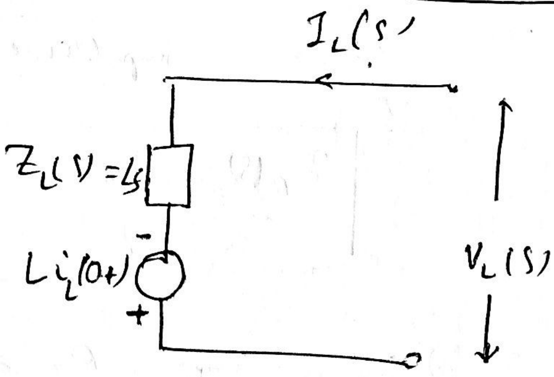


(a) L with initial current $i_L(0+)$



(b) X'frm representation

(c) capacitance :-

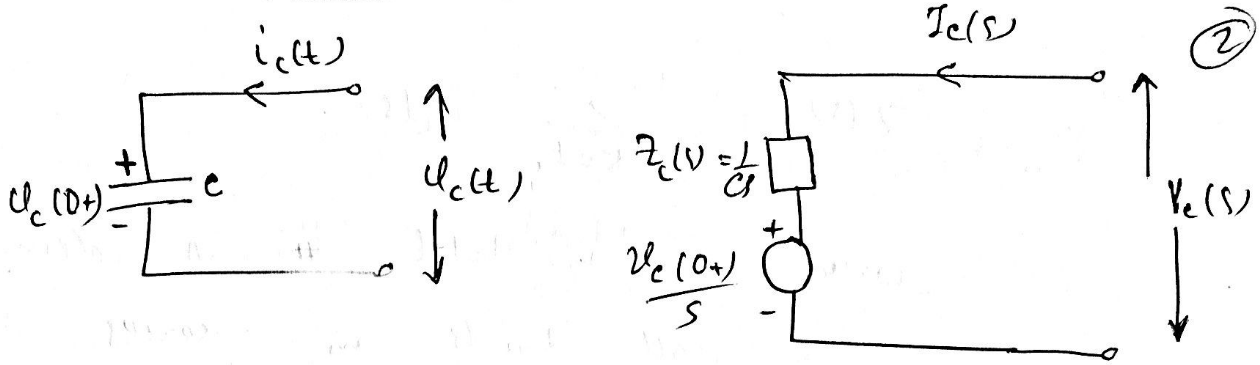
$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt$$

or $V_c(s) = \frac{1}{C}\left[\frac{I_c(s)}{s} + \frac{q(0+)}{s}\right]$

where $q(0+)$ is initial charge across capacitor

&

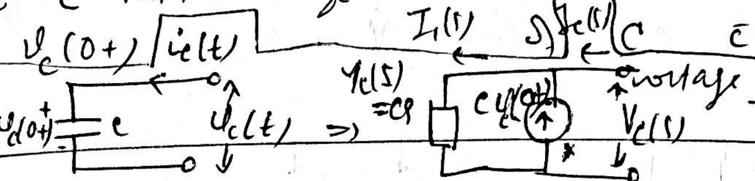
$$\frac{q(0+)}{C} = v_c(0+)$$



a) T-D. representation of C with initial voltage

(b) Xform representation of C with initial voltage $v_c(0+)$. from $i_c(t) = C\frac{dv}{dt}$

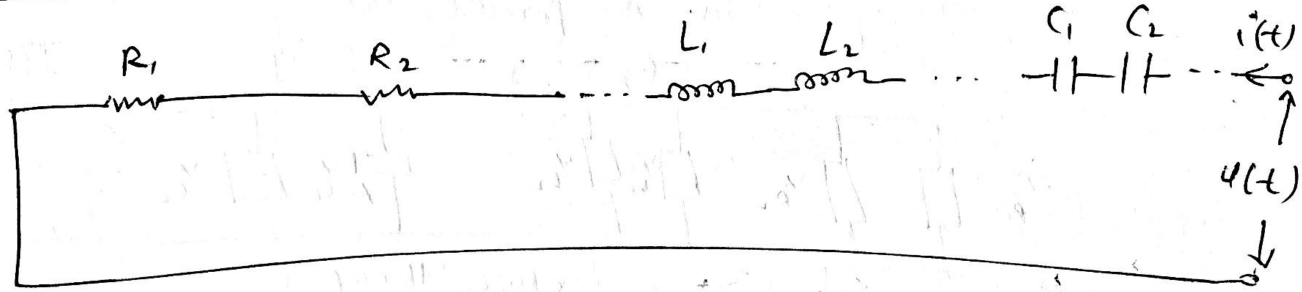
IIIly, $Y_c(s) = Cs$ &



* I-src

Series & Parallel combination of Elements :-

(i) Series combination :-



here,

$$v(t) = v_{R_1} + v_{R_2} + \dots + v_{L_1} + v_{L_2} + \dots + v_{c_1} + v_{c_2} + \dots \quad (1)$$

L. Xfrm gives

$$V(s) = V_{R_1}(s) + V_{R_2}(s) + \dots + V_{L_1}(s) + V_{L_2}(s) + \dots + V_{C_1}(s) + V_{C_2}(s) + \dots \quad (2)$$

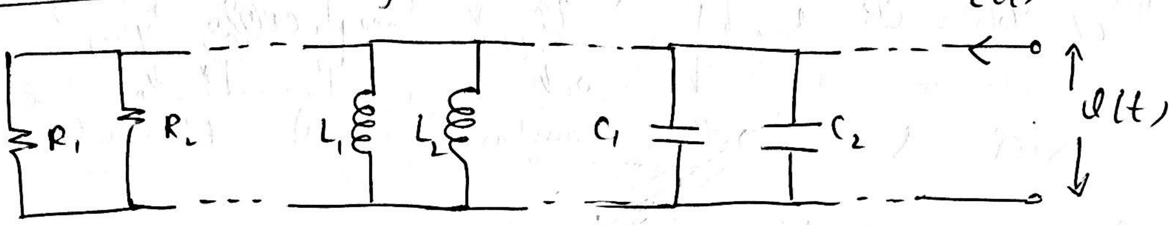
$\div$ing B/S of (1), by $I(s)$, we have

$$Z(s) = Z_{R_1}(s) + Z_{R_2}(s) + \dots + Z_{L_1}(s) + Z_{L_2}(s) + \dots + Z_{C_1}(s) + Z_{C_2}(s) + \dots \quad (3)$$

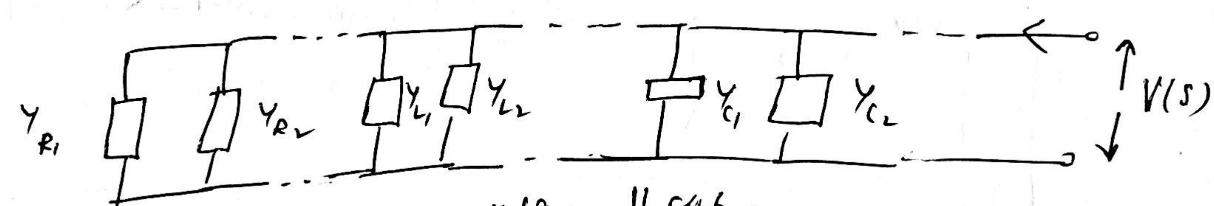
or $$Z(s) = \sum_{k=1}^{n} Z_k(s)$$.

where n is total # of elements of all kinds in series

(ii) <u>Parallel combination of elements</u> :-



(a) A parallel ckt



(b) Xform ‖ ckt.

Here from ckt (a),

$$i(t) = i_{R_1}(t) + i_{R_2}(t) + \dots + i_{L_1}(t) + i_{L_2}(t) + \dots + i_{C_1}(t) + i_{C_2}(t) + \dots \quad \text{— (1)}$$

Taking L-xform of (1), we have

$$I(s) = I_{R_1}(s) + I_{R_2}(s) + \dots I_{L_1}(s) + I_{L_2}(s) + \dots + I_{C_1}(s) + I_{C_2}(s) + \dots \quad \text{— (2)}$$

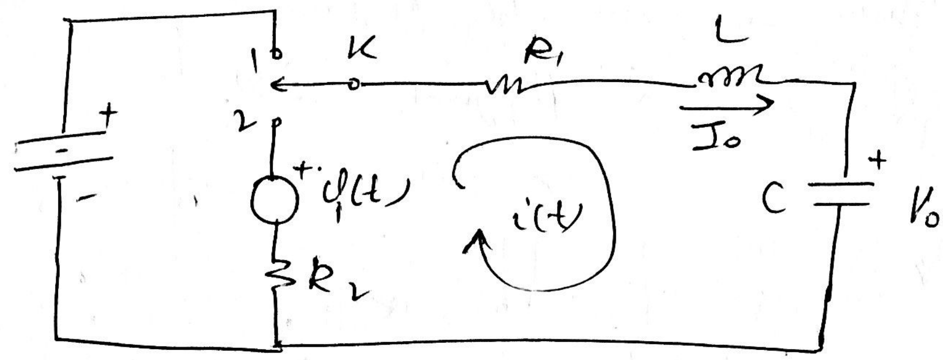
Dividing B/S of (2) by $V(s)$, we get

$$Y(s) = Y_{R_1}(s) + Y_{R_2}(s) + \dots Y_{L_1}(s) + Y_{L_2}(s) + \dots + Y_{C_1}(s) + Y_{C_2}(s) + \dots$$

or $$Y(s) = \sum_{k=1}^{n} Y_k(s)$$

where n is # of elements in parallel.

Example :-



K is moved from p1 to p2 at $t = 0$.

At $t = 0-$, $i_L(0-) = I_0$

& $v_c(0-) = V_0$.

At $t = 0+$, voltage src $v_1(t)$ & resistor R_2 get connected to the R-L-C ckt.

find $I(s)$ & $i(t)$

Soln :- ~~V(s)~~

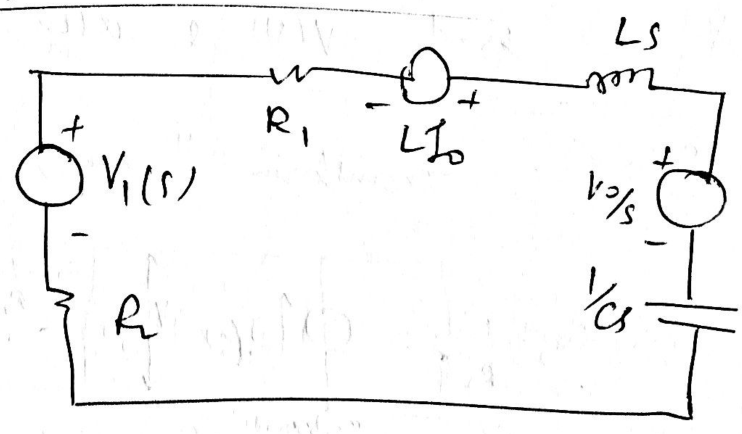


Fig. xform ckt.

Total transform voltage in N/W is given by

$$V(s) = V_1(s) + LI_0 - \frac{V_0}{s}$$

Total xfrm impedance is

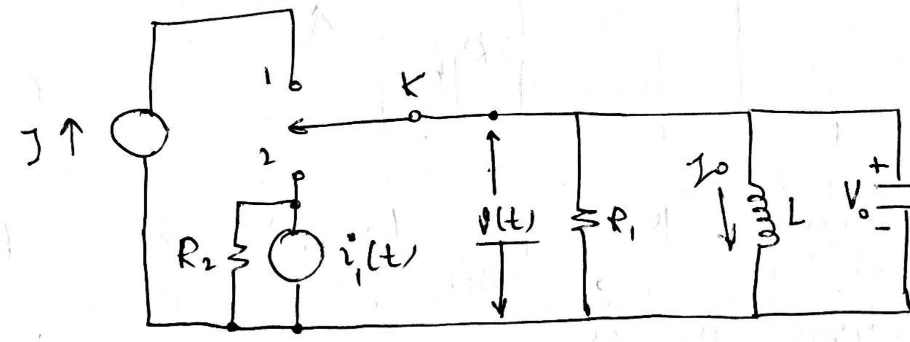
$$Z(s) = R_1 + R_2 + Ls + \frac{1}{Cs}$$

∴ xfrm current is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V_1(s) + LI_0 - V_0/s}{R_1 + R_2 + Ls + \frac{1}{Cs}}$$

This eq can be expanded using p.f. method & on $\mathcal{L}^{-1}$ it gives $i(t)$.

(#)



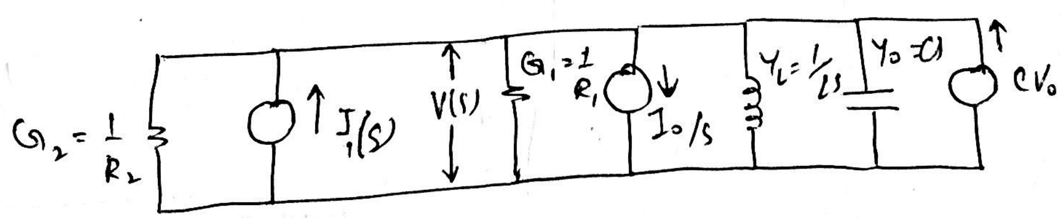
K is moved from p1 to p2 at $t = 0$.

$i_L(0-) = I_0$ & $V_c(0-) = V_0$.

At $t(0+)$, $i_1(t)$ & R gets connected to the $R_1 - L - C \parallel$ ckt.

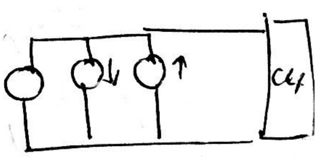
Find $V(s)$ & $v(t)$ across R_1-L-C $\parallel$ ckt.

Soln :- Transform N/w can be drawn as :-



Total Xform current in the N/w is :-

$$I(s) = I_1(s) - \frac{I_0}{s} + CV_0$$



Total Xform admittance is given by :-

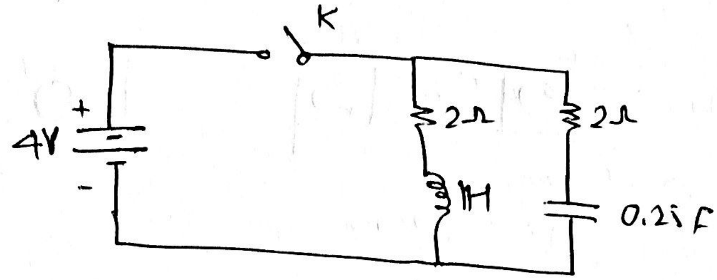
$$Y(s) = G_1 + G_2 + \frac{1}{Ls} + Cs$$

$$\therefore \quad V(s) = \frac{I(s)}{Y(s)} = \frac{I_1(s) - I_0/s + CV_0}{G_1 + G_2 + \frac{1}{Ls} + Cs}$$

$\mathcal{L}^{-1}\{V(s)\} \rightarrow v(t)$

(#) In given N/w, K is opened at t = 0. Draw transform N/w & find current $i(t)$ in the loop. (4)

~~Solution~~ :-

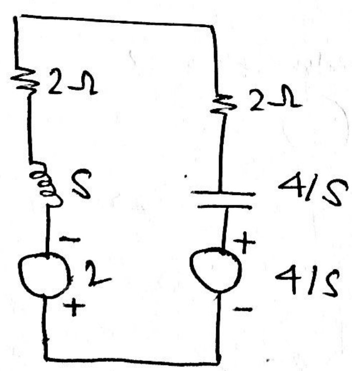


Soln :- Under S.S.

$$i_L(0-) = 2A$$

$$\& \ v_c(0-) = 4V \text{ d.c.}$$

$\therefore$ transform N/w is

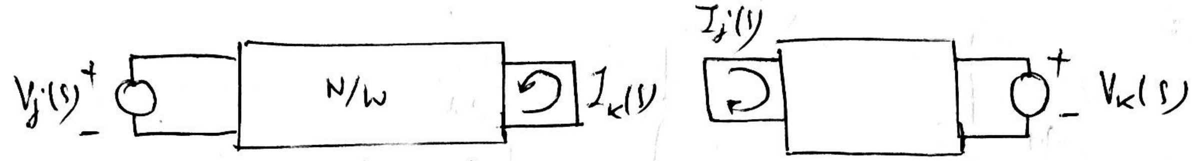


$$I(s) = \frac{V(s)}{Z(s)} = \frac{2 + 4/s}{2 + 2 + s + 4/s} = \frac{2(s+2)}{(s+2)^2} = \frac{2}{s+2}$$

is $\boxed{i(t) = 2e^{-2t} \cdot u(t)}$

Reciprocity Theorem :-

Given 2 N/W's (a) & (b), we have if



For a ~~symmetrical~~ reciprocal N/W N/W,

$$\frac{V_j(s)}{I_k(s)} = \frac{V_k(s)}{I_j(s)}$$

thus, $Z(s)$ matrix i.e. impedance matrix is symmetrical.

Further if $V_j(s) = V_k(s)$, then $I_k(s) = I_j(s)$.

→ thus for a reciprocal N/W, we infer that the ratio of response Xform to the excitation Xform remains the same when we interchange the positions of response & excitation in the N/W.

→ this is principle of reciprocity.

→ such N/W's are known as reciprocal N/W's.

→ A N/W is reciprocal if :-

i) the N/W is initially relaxed.

ii) only linear elements are present in the N/W.

iii) dependent or controlled sources even if linear must be absent.